Coupling two cell-centered finite volume schemes for solving anisotropic diffusion and fluid dynamics on unstructured moving meshes

Pascal Jacq, P.H. Maire, Jean Claudel, Remi Abgrall

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P. Jacq [a,*], P.-H. Maire [a], J. Claudel [a], R. Abgrall [b]

[a] CEA - Cesta
BP 2 - 33114 Le Barp, France

[b] INRIA Bordeaux - Sud-Ouest
200 avenue de la Vieille Tour - 33405 Talence Cedex, France

*pascal.jacq@cea.fr

Abstract. *In our work we intend to model the physical phenomenons occuring during the atmospheric re-entry of a spacecraft at a low altitude. We developped a cell-centered finite volume scheme for solving anisotropic diffusion on unstructured moving meshes in order to model the thermal transfers occuring in the re-entry vehicule. We also developped a cell-centered finite volume scheme for solving the Navier-Stokes equations in order to model the hypersonic fluid flow around the vehicle. In this talk we will focus on the modelisation of the ablation phenomenon which introduce a coupling between the flow and the thermal transfers.*

Keywords: second-order finite volume; unstructured meshes; anisotropic diffusion; ALE framework; fluid dynamics; coupling.

1 INTRODUCTION

In this talk we describe two cell-centered finite volume schemes developed in an Arbitrary Lagrangian Eulerian (ALE) framework for solving respectively anisotropic diffusion equation and Navier-Stokes equations. We then describe an ablation model that is used as a boundary condition in the two schemes to introduce the coupling between the different phenomenons.

2 DIFFUSION EQUATION

Let us start by recalling the diffusion equation that model the heat transfers in solids.

$$\rho C_v \frac{\partial T}{\partial t} + \nabla \cdot \boldsymbol{q} = \rho r. \tag{1}$$

T is the temperature field we want to determine, ρ is a positive real valued function which stands for the mass density of the material. The source term, r, corresponds to the specific heat supplied to the material and C_v denotes the heat capacity at constant volume. Equation (1) is a partial differential parabolic equation of second order for the temperature T, wherein the conductive flux, $\boldsymbol{q}$, is defined according to the Fourier law:

$$\boldsymbol{q} = -\boldsymbol{K}\nabla T. \tag{2}$$

The second-order tensor $\boldsymbol{K}$ is the conductivity tensor which is an intrinsic property of the material under consideration.

To solve this equation we develop a cell-centered finite volume scheme called CCLAD based on the work of Maire and Breil described in [1, 2]. For two-dimensional geometry, we define two half-edge temperatures and two half-edge normal fluxes per cell interface using the partition of each cell into sub-cells. The half-edge normal fluxes are then expressed in terms of half-edge temperatures and cell temperature using a local variational formulation. Finally the half-edge temperatures surrounding a node can be eliminated by solving a small linear system which is obtained by enforcing the continuity condition of the normal heat fluxes across the sub-cells interfaces surrounding this node.

3 NAVIER-STOKES EQUATIONS

Let us continue by recalling the Navier-Stokes equations that model the fluid flow around our re-entry vehicle.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0, \tag{3}$$

$$\rho \left[\frac{\partial V}{\partial t} + (V \cdot \nabla) V \right] = -\nabla p + \mu \left[\nabla^2 V + \frac{1}{3} \nabla (\nabla \cdot V) \right] + \rho f, \tag{4}$$

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot [\, (\rho e + p) V \,] = \nabla \cdot (\boldsymbol{\tau} \cdot V) + \rho f \cdot V + \nabla \cdot (\lambda \nabla T). \tag{5}$$

ρ is the density of the fluid, V its velocity, e its internal energy and T its temperature. p stands for the pressure of the fluid and is given by an equation of state. The fluids we are working with are real gases.

Once again we developed a cell-centered finite volume scheme for solving theses equations. The diffusive part of theses equations are solved using the CCLAD scheme presented above while the remaining parts are solved using a classical finite volume scheme using a MUSCL approach.

4 ABLATION MODEL

Let us consider the ablation phenomenon. The heat shields of re-entry vehicle are mainly made of Carbon. In a first model we consider that the sublimation of the C_3 species is the preponderant reaction of the ablation: $3C \longrightarrow C_3$ as presented in [5].

$$\dot{m}_{sub} = \alpha_3 \sqrt{\frac{M_{C_3}}{2\pi RT}} (\bar{p}_{C_3} - p_{C_3}), \tag{6}$$

where $\dot{m}_{sub}$ is the mass ablation rate. α_3 is the accommodation coefficient of the C_3 species, M_{C_3} is the molar mass of C_3, $R = 8.3143$J/(K.mol) is the perfect gas constant, p_{C_3} and T are the pressure and temperatures at the interface.

$\bar{p}_{C_3} = 2.82110^5 A_3 T^{n_3} \exp\left(-\frac{E_3}{T}\right)$ is the saturated vapor pressure of C_3 where $A_3 = 4.310^{15}$, $n_3 = -1.5$, $E_3 = 97597.0K$.

This ablation model is introduced in the boundary conditions of the Navier-Stokes and diffusion schemes in order to couple the physics through the temperature field. We remark that this model introduce the motion of the ablating interface which is taken into account by the ALE framework of the numerical schemes.

5 CONCLUSIONS

We developed two cell-centered finite volume schemes to solve anisotropic diffusion equation and Navier-Stokes equations. We also presented a simple ablation model. The interest of this talk lies in the presentation of the coupling of theses equations through the ablation model.

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